

Appendix 3: Significance of a Chi-Square Statistic

For 30 or fewer degrees of freedom, an exact series expansion is used; otherwise the Peizer-Pratt approximation is used.

Notation

The following notation is used in this appendix:

X	Value of the chi-square statistic
k	Degrees of freedom
Q	Significance level (right-tail probability)

Computation

- If $X \leq 0$ or $k < 1$,

$$Q = 1$$

- If $k = 1$,

$$Q = 2Q_N(\sqrt{X})$$

where $Q_N(\sqrt{X})$ is the standard normal one-tailed significance probability.

- For $k \leq 30$, an exact series expansion is used (Abramowitz and Stegun, 1965, eqs. 26.4.4 and 26.4.5)

$$Q = \begin{cases} 2Q_N(\sqrt{X}) + R\sqrt{\frac{2}{\pi}} \exp\left(\frac{-X}{2}\right) & k \text{ odd} \\ \exp\left(\frac{-X}{2}\right) \times (1 + R) & k \text{ even} \end{cases}$$

where

$$R = \begin{cases} \sum_{r=1}^{(k-1)/2} \frac{X^{r-1/2}}{1 \cdot 3 \dots (2r-1)} & k \text{ odd} \\ \sum_{r=1}^{(k-2)/2} \frac{X^r}{2 \cdot 4 \dots 2r} & k \text{ even} \end{cases}$$

- If $k > 30$, the Peizer-Pratt approximation is used (Peizer and Pratt, 1968, eq 2.24a).
- If $X \geq 150$,

$$Q = 0$$

otherwise

$$Q = Q_N(Z)$$

where

$$Z = \begin{cases} \left(-\left(\frac{1}{3} + \frac{0.08}{k} \right) \right) / (\sqrt{2k-2}) & \text{if } X = k-1 \\ \left(d \sqrt{(k-1) \log\left(\frac{k-1}{X} \right) + X - (k-1)} \right) / |X - (k-1)| & \text{if } X \neq k-1 \end{cases}$$

where

$$d = X - k + 2/3 - 0.08/k$$

- If $Z < 0$,

$$Q = 1 - Q_N(Z)$$

References

Peizer and Pratt (1968)